

## 1. Introduction

Predicting performance of mass produced aircraft has become a nearly exact science, within the limits of variations in component quality and material properties, as well as changes in weather. However predicting the outdoor free flight model airplane performance (read 'flight duration') feels anything but exact even to skilled old hands at the game. This article is aimed at removing at least a bit of the mystery and clarifying what's really important if maximum duration is your goal.

## 2. The Basic Duration Formula:

Let's assume that you have two models of the same real aircraft, built to exactly the same scale, have been flight trimmed to fly equally well and are flying in the same still air. Question: To a reasonable degree of accuracy, which remaining parameters define how long each model will stay up? Also, what are the relative influences of those crucial parameters?

A reasonably accurate answer lies in the "Free Flight Duration Formula" which is derived by assuming a constant velocity flight\* and then setting:

- 1) overall lift equal to gross weight of the model plane,
- 2) total effective propulsion energy equal to total energy dissipated by drag during the flight,
- 3) flight duration equal to distance travelled divided by velocity.

The above formulation produces 8 equations that can be solved simultaneously to yield "The Duration Formula", a single equation for flight duration,  $T$ :

$$T = \frac{K_r \eta_p C_l^{1.5} \sqrt{\rho_{air}/2}}{C_d} * \frac{W_r / W_{gross}}{\sqrt{W_{gross} / A_{wing}}} \quad (1)$$

Where:  $K_r$  = Specific energy of the motor rubber

$\eta_p$  = average propeller efficiency

$C_l$  = overall lift coefficient of model referenced to wing area

$\rho_{air}$  = air density

$C_d$  = overall drag coefficient of model referenced to wing area

$W_r$  = weight of rubber motor

$W_{gross}$  = total weight of model and rubber motor

$A_{wing}$  = wing area

$W_r / W_{gross}$  = "Power Loading"

$W_{gross} / A_{wing}$  = "Wing Loading"

Now, since both models have the same rubber, prop, and aerodynamics, the first large fraction in equation 1 as well as the wing area are the same for each. So to change duration your only "levers" are rubber weight and model gross weight. Duh! You already knew that. But the formula

does at least quantify the effect of any changes you make in those weights. It also identifies the effects of changes in rubber material, prop design, or coefficients of lift and drag.

\*Note: In situations where the model is set up to provide a fast climb followed by a slower glide, the actual duration will be less than the result of eqn 1 because drag increases with the square of velocity, thereby dissipating propulsion energy faster than in a slower constant velocity flight.

### 3. Converting to a formula that can be used to make a simple graph:

Plugging numbers into Eqn 1 is a little messy because any change in rubber weight,  $W_r$ , changes gross weight,  $W_{gross}$ , as well. Applying some more algebra will get us a more useful form of Eqn 1:

$$T = K \frac{\left\{ \frac{R}{(R+1)^{1.5}} \right\}}{\sqrt{W_o / A_{wing}}} \quad (2)$$

Where:  $K = K_1 * \frac{K_r \eta_p C_l^{1.5} \sqrt{\rho_{air}/2}}{C_d} \approx \underline{285}$ , a value developed for typical well designed, low drag FAC scale models by AMA Hall of Famer, Jim Alaback. For high-drag WW1 biplanes with lots of rigging,  $K$  should be reduced to 230 or less.

$K_1$  = a factor to reconcile the mixed units of equation 2, since US modelers commonly express weights in grams and dimensions in inches.

$W_o$  = Dry weight of complete model, not including rubber motor (gm).

$R = W_r / W_o$  = “Dry weight power loading” of model, not including motor (gm/gm).

$W_o / A_{wing}$  = “Dry weight wing loading” of model, not including motor (gm/in<sup>2</sup>).

Figure 1 on the next page presents a graph of equation 2 for a typical range of ‘dry weight’ wing and power loadings. It provides us with a tool for predicting duration as you proceed from choosing a model subject and then do weights analysis, material selection, construction and motor sizing.

### 4. Using the graph: a personal example:

As an example let’s look at two versions of my “**Twice Mooney**” Miles M5 “**Sparrowhawk**”, a 1930’s British racing plane. **M5-1** was built in 2000 and covered in non-scale yellow tissue with fixed gear, a 9.5’ Peck plastic prop, and a dry weight (including nose weight) of 43.6 gm. It flew well, garnering a couple of Kanones in local FAC scale events. Deciding to upgrade for the 2006 FAC Nats, I changed up to **M5-2**, enlarging the stabilizer, repainting the original plane with proper cream colored floral spray, adding vinyl lettering and changing to breakaway magnet-attached wheel pants. See photos of both models in the Appendix A. Here is a table comparing the two versions:

Model	Wing Area ( Sq. In.)	Bare dry Weight (gm)	Balanced Dry Wt = $W_o$ (gm)	Rubber Wt = $W_r$ (gm)	$W_r / W_o$	$W_o / A_{wing}$	Duration (est. sec)
M5-1	130	43.7	44.0	13.0	0.295	0.34	98
M5-2	130	53.8	62.0	19.0	0.307	0.48	84

The data for the two versions of my M5 are entered as red stars on Figure 1 where the resulting estimated durations can be read on the vertical axis of the graph.

**Figure 1:**

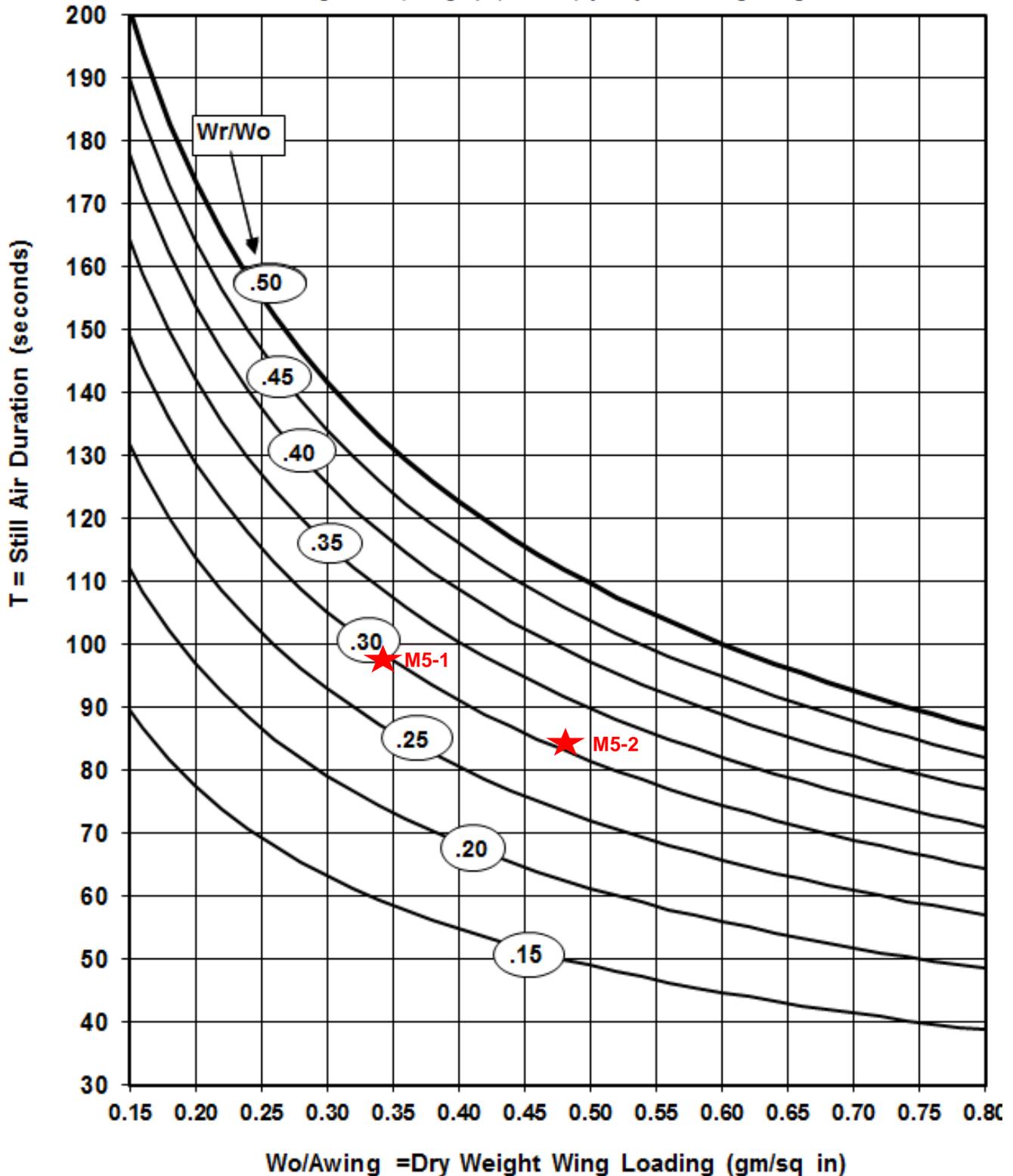
**Still Air Duration vs  $W_r/W_o$  and  $W_o/A_{wing}$**

**Duration Formula:  $T = K * W_r * A_{wing}^{0.5} / W_{gross}^{1.5}$**

Where:  $W_r$  = motor weight (gm),  $W_o$  = Total model wt., less motor (gm)

$W_{gross}$  = total wt of model including motor(gm),  $A_{wing}$  = wing area (sq. in)

$K = 285$  for low drag model (this graph); Multiply T by 0.8 for high drag model



## **5. Discussion of the graph's results:**

Comparing the two models in Figure 1, you could argue that M5-2 could achieve the duration of M5-1 by simply adding more rubber weight, i.e. increasing its  $W_r / W_o$  from 0.307 to 0.39. (In fact I use 40% motors in most of my current models). In this case however there is a hidden price that was paid in the first flights of M5-2. Since I hadn't moved the motor peg for the new version, its aft-biased heavier motor and larger stabilizer necessitated adding significantly more nose ballast for proper trim. Even with both M5-1 and M5-2 having roughly 30% motors, the gross weight ratio of the two models ended up as  $W_{gross-2} / W_{gross-1} = 1.42$ . Since heavier planes must fly faster to maintain the higher lift required in flight, the kinetic energy ( $Mass * V^2$ ) of the M5-2 was over twice that of M5-1! Bottom line: the heavier M5-2 flew much faster, resulting in much harder landings (crashes?), lots of road rash, and occasional partial re-kitting. Its grand finale was a significant dismemberment at the 2006 FAC Nats! Of course, even light-weight models can have hard downwind landings on windy days.

A second problem with increasing motor weight to increase power loading of overly heavy models, is the difficulty of cramming a large motor into the confines of a typically narrow scale model fuselage. Note that figure 1 shows that for any given power loading,  $W_r / W_o$ , duration increases significantly with just a 15% reduction in both dry weight and resultant wing loading. Play around by entering your models' power and wing loadings to find your own estimated durations.

## **6. Some weight reduction ideas:**

Since my Sparrowhawk Saga, within the limits of maintaining structural and scale integrity, I've made a near fetish of weight reduction. Even FAC non-scale planes have "original plan structure" rules that force us to pay careful attention to material selection, décor, and other details so that we end up with competitively lightweight models. In recent builds, my Flying Aces Moths have had dry weights of 27-29 grams and my Jimmie Allen Skokies typically weigh in at 35-37 grams.

Weight reduction steps included:

1. Replacing the Peck plastic props with 55% lighter molded balsa props and lightweight hubs in a reworked Gizmo Geezer assembly.
2. Minimizing tail weight bias by moving the motor peg forward, applying little or no dope on the tail feathers, and using very small hardware for the stab DT line conduit and decalage adjustment. These steps eliminated the need for nose ballast even when using my light balsa props.
3. Lightening anything not specified on the plan- For widow glazing I use 1 mil material for the sides and 3 mil for the front. Wing and stab hold-downs are mini orthodontic rubber bands.
4. Within the appearance rules, lightening wheels and other details; my proper size Skokie wheels use laminated light balsa construction with swiss-cheese cutouts in the invisible inner layers.
5. Using only tissue for decor and applying only two light coats of Krylon on the wings and fuselage.

See the Appendix B for pictures of recent FA Moth and Skokie models.

For scale models, I have changed to using mostly chalked tissue for scale-like colors. It seems that whenever I paint a model, right away its weight increases by 20+ %. Since most of my scale ships are flown in mass launches, their props are still the tough old plastic Pecks, scraped to take off a little weight.

All my lighter current models, needing less lift, are set up with long, skinny motors driving relatively high pitch props which, *when the planes are well trimmed*, produce slow steady climbs and gentle glides. Motor runs are often in the range of 1.5 + minutes.

See the appendix B for photos of my recent FA Moth and Skokie models.

#### **6. My next article:**

My main mathematical method for maintaining weight consciousness and reducing weight during building, is contained in an Excel 'model-airplane-design' spreadsheet that I developed for an article in the 2008 NFFS Symposium. In a new tutorial article next month I'll provide a step-by-step set of instructions for using this tool. Based on your input data (which can be upgraded as your model takes shape), this spreadsheet will perform weight and balance calculations, determine tail volume, recommend CG locations, find required nose or tail ballast, compute power and wing loadings, calculate estimated flight duration, and suggest appropriate motor setups for your desired power loading ( $W_r / W_o$ ). Your feedback about this article and my upcoming design spreadsheet piece will be most welcome.

My email address is: [gmansfield75@gmail.com](mailto:gmansfield75@gmail.com)

#### **7. A final suggestion:**

Click the "Tech" tab of the Scale Staffel website to find an excellent article by retired British aeronautical engineer and long-time modeler, George Seyfang. In it he provides wind tunnel tested methods for significantly increasing a model's lift/drag ratio by adding 'Gurney flaps' and T strips to both the wing and tail. Look back to equation 1 on page 1 of this present paper to see the improvements these increases in  $C_l / C_d$  might produce in your model planes' flight times. His large home-built wind tunnel can be seen under the "Tools" tab of the website.

**Appendix A-Two versions of a 'Miles M5 Sparrowhawk' Scale Model:**

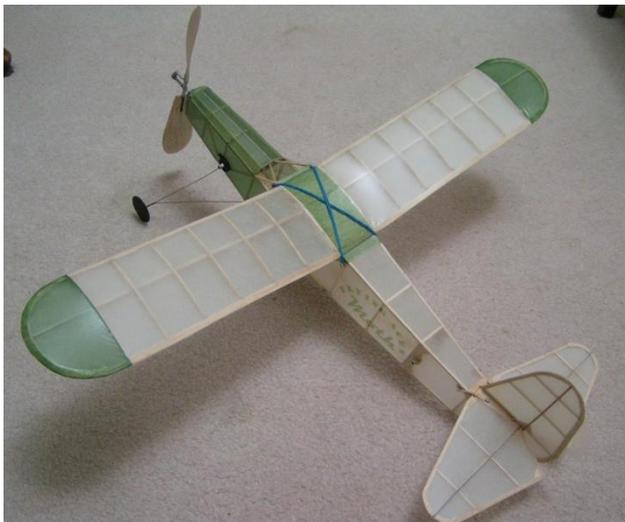


**M5-1 (2000)**

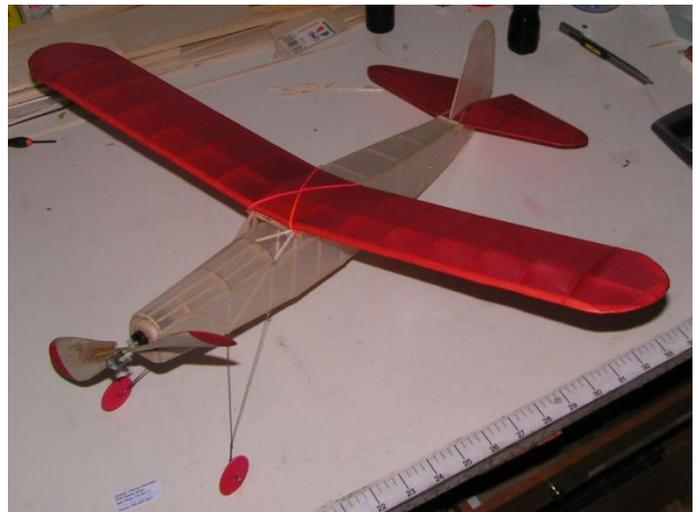


**M5-2 (2006)**

**Appendix B-Recent Versions of 'Flying Aces Moth' and 'Jimmie Allen Skokie' Models:**



**Moth-2012**



**Moth-2015**



**Skokie-2016**